Non-Fermi liquid behavior of the electrical resistivity at the ferromagnetic quantum critical point

D. Bodea[†], M. Crisan, I. Grosu, and I. Tifrea[‡]
Department of Theoretical Physics, University of Cluj, 3400 Cluj-Napoca, Romania

We propose a model for the non-Fermi behavior in the proximity of the quantum phase transition induced by the strong polarization of the electrons due to local magnetic moments. The self-consistent Renormalization Group methods have been used to calculate the temperature dependence of the electrical resistivity and specific heat. The $T^{5/3}$ dependence of resistivity and the $T \ln T$ dependence of the specific heat show that the magnetic impurities drive a ferromagnetic quantum phase transition and near the critical point the system present a non-Fermi liquid behavior. The model is in good agreement with the experimental data obtained for Ni_xPd_{1-x} alloy.

I. INTRODUCTION

Deviations from the standard Fermi liquid description in heavy fermion systems (HFS) have been associated to the proximity of the quantum phase transition (QPT). Defined as phase transitions at $T = 0$, QPT's are usually driven by quantum fluctuations controlled by a non-thermal parameter, namely by impurities, pressure or magnetic fields.[1] Most of these low temperature magnetic QPT's are from paramagnetic to antiferromagnetic state, but recently HFS undergoing a paramagnetic to ferromagnetic phase transitions were identified. Example of HFS which are close to ferromagnetic order are $Th_{1-x}U_xCu_2Si_2$,[2] MnSi,[3] and Ni_xPd_{1-x}.[4] The itinerant-electron ferromagnetic state in these materials is induced by pressure (MnSi) or impurities ($Th_{1-x}U_xCu_2Si_2$ and Ni_xPd_{1-x}).

$Th_{1-x}U_xCu_2Si_2$ for concentrations $x \geq 0.15$ presents ferromagnetic order. Experimental data in these compounds show a non-Fermi liquid behavior for $C/T \propto \ln T$ and a critical temperature, T_c, as low as 12K for the $x = 0.15$ sample.[2] However, the magnetic susceptibility data present a field dependance even at temperatures bellow T_c, a behavior characteristic for weak itinerant ferromagnets.

In the case of MnSi experimental results showed the existence of a first-order phase transition induced by pressure, the critical temperature decreasing towards absolute zero at a pressure value $p_c \simeq 14.6$ kbar. For $p > p_c$ a non-Fermi liquid behavior of the resistivity as function of temperature is reported, $\rho \sim T$.[3] The phenomenological theory [3] based on the interaction of the electrons with the overdamped spin fluctuations at low energies, can explain the occurrence of the non-Fermi state. The quantum transport anomalies of the itinerant-electron ferromagnetic state have been discussed also by Belitz, Kirkpatrik, Narayanan and Vojta (BKNV) [5], the non-Fermi behavior of the system being proved based on the scaling approach. The model from Ref. [5] predicts a first order phase transition and it is in an excellent agreement with the QPT driven by pressure in MnSi [3].

Ni_xPd_{1-x} at a Ni concentration $x_c \cong 0.025$ presents ferromagnetic order assumed to occur in the itinerant electron system of Pd atoms due to a strong polarization of these electrons by magnetic impurities (in this case Ni). The transition critical temperature depends on the Ni concentration as $(x - x_c)^{3/4}$ in the critical region. In the same region of the phase diagram, experimental data reveal a $T^{5/3}$ dependance of the relative resistivity, $\Delta\rho$, and a $T \ln T$ dependance of the specific heat, $C(T)$.[4] All these experimental data clearly identify the non-Fermi character of the electronic excitations close to the quantum critical point (QCP). These results were successfully described in terms of phenomenological spin-fluctuation models.[6]

In this work we extend the Doniah-Wohlfarth model,[7] proposed for the explanation of the itinerant electron ferromagnetic state driven by impurities in Ni_xPd_{1-x} compounds, to the critical region and we calculate using renormalization group methods the temperature dependence of the resistivity and the specific heat. We will apply the Hertz-Millis [8, 9] version of the Renormalization Group method (RNG) to the Doniah-Wohlfarth model taking into consideration the quantum effects at finite temperature and extract the effects of the spin fluctuations on the system. The self-consistent renormalization group theory, given by Moriya [10] will be used to calculate the temperature dependence of the electrical resistivity.

II. MODEL

We consider that the fluctuations in the magnetization in the critical region are given by the action:

$$S_{eff} \;=\; S^{(2)}_{eff} \;+\; S^{(4)}_{eff}\,, \tag{1}$$

where

$$S^{(2)}_{eff} \;=\; \frac{1}{2}\sum_{q}\; \chi^{-1}(q)\;|\phi(q)|^2 \tag{2}$$

and

$$S^{(4)}_{eff} \;=\; \frac{u}{4}\sum_{q_1}\ldots\sum_{q_4}\; \phi(q_1)\ldots\phi(q_4)\;\delta(q_1+\cdots+q_4)\,. \tag{3}$$

Here we introduce the notation $q = (\mathbf{q}, \omega_n)$, ω_n being the bosonic Matsubara frequencies, and

$$\sum_q \; = \; k_B T \sum_n \int \frac{d^d \mathbf{q}}{(2\pi)^d} \; .$$

In Eqs. (2)-(3) $\chi(q)$ is the fluctuation propagator and u is the coupling constant.

In the following we consider that the spin impurity has a very strong polarization effect on the electrons and at a critical concentration $x = x_c$ a new phase, which is in fact a ferromagnetic phase, can be reached. This model has been proposed first by Donniach and Wolhfarth [7] using a single impurity approximation. The susceptibility of the polarized electrons was given (see Ref. [7]) as

$$\chi(\mathbf{q}, \omega) \; = \; \frac{\chi_0(\mathbf{q}, \omega)}{1 - \left(I + 2\frac{J^2 R'}{JR - \omega} \right) \; \chi_0(\mathbf{q}, \omega)} \; , \qquad (4)$$

where $\chi_0(\mathbf{q}, \omega)$ is the susceptibility of the electronic system. In Eq. (4) J is the exchange interaction between electrons and localized spins and I is the interaction between electrons. The parameters R and R' have been calculated as

$$R \; = \; \frac{1}{N} \; \sum_k (n_{k\downarrow} - n_{k\uparrow}) \qquad (5)$$

and

$$R' \; = \; -x \; \langle S^z \rangle \; , \qquad (6)$$

x being the impurity concentration of the magnetic moments with spin $\mathbf{S}$.

The dynamic susceptibility $\chi_0(\mathbf{q}, \omega)$ has the form [10]

$$\chi_0^{-1}(\mathbf{q}, \omega) \; \simeq \chi_0^{-1}(0,0) \left[1 \; - \; Dq^2 \; + \; iC\frac{\omega}{q} \right] \; , \qquad (7)$$

where D and C are constants. We approximate $\chi(\mathbf{q}, \omega)$ from Eq. (4) as

$$\chi(\mathbf{q}, \omega) \; = \; \frac{1}{\chi_0^{-1}(\mathbf{q}, \omega) - I - 2\frac{JR'}{R}} \; , \qquad (8)$$

a results which based on Eq. (4) can be written as

$$\chi(\mathbf{q}, \omega) \; = \; \frac{1}{\delta_0(x) + aq^2 - i\frac{\omega}{\Gamma q}} \; , \qquad (9)$$

where a and Γ are constants, and $\delta_0(x)$ is given by

$$\delta_0(x) \; = \; \chi_0^{-1} \; - \; I \; - \; 2\frac{JR'}{R} \; . \qquad (10)$$

The parameter $\delta_0(x)$, with a linear dependance on the impurity concentration, measures the distance from the QCP.

The model is valid only for systems in which the local moments give a strongly polarization of the itinerant electrons and this is typically for metals as Pd which is paramagnetic and the Stoner criterium cannot be satisfied only due to the electron-electron interaction. For this particular system we do not expect localization effects, excepting the case of nonmagnetic impurities, but this cannot drive the system in the ordered phase.

This model, described by Eqs. (1), (2) and (9), can be treated using the Renormalization Group method, in the version proposed by Hertz [8] and Millis [9]. Following Refs. [8, 9] we perform the standard scaling $k \to k'/b$, $\omega_n \to \omega_n'/b^z$, z being the dynamic critical exponent. We obtain the following flow equations:[11]

$$\frac{dT(l)}{dl} \; = \; zT(l) \; , \qquad (11)$$

$$\frac{d\Gamma(l)}{dl} \; = \; (3 - z)\Gamma(l) \; , \qquad (12)$$

$$\frac{d\delta(l)}{dl} \; = \; 2\delta(l) \; + \; 2(n+2) f_1 u(l) \; , \qquad (13)$$

$$\frac{du(l)}{dl} \; = \; [4 - (d+z)]u(l) \; - \; (n+8) f_2 \; u^2(l) \; , \qquad (14)$$

where d is the spatial dimension, f_i are functions characteristic for the model (see Ref. [9]) and $l = \ln b$ is the scaling variable. Additionally, the system free energy will scale as:

$$\frac{dF(l)}{dl} \; = \; (d+z)F(l) \; + \; f_3 \; , \qquad (15)$$

f_3 being again a characteristic function of the model.

III. SPECIFIC HEAT AND RESISTIVITY

The evaluation of the renormalized free energy $F[T(l)]$ based on Eq. (15) will lead to the temperature dependence of the specific heat which by definition can be calculated as the second derivative of the free energy with respect to the temperature, $C(T) = -T(\partial^2 F/\partial T^2)$. In general there will be two distinct contributions to the renormalized free energy, associated to a quantum domain, $T(l) \ll 1$, and to a classical one, $T(l) \gg 1$. For more details on the calculation of the free energy see Refs. [9, 11]. However, as the form of the system susceptibility match the corresponding form for a ferromagnetic system, if we consider the $d = 3$ case, the specific heat is obtained as

$$C(T) \; = \; \gamma_0 T \; + \; \gamma_1 T \; \ln T \; , \qquad (16)$$

γ_0 and γ_1 being constants, a result which clearly show that the behavior of the considered system is non-Fermi, as corrections to the linear temperature dependance of the specific heat are logarithmic. This result is in agreement with the experimental data presented in Ref. [4].

We have to mention that a similar behavior was obtained using RG method for a system in the proximity of the Lifshitz quantum critical point.[12] Recent experimental data in silicon MOSFETs[13] suggested a QPT to a ferromagnetically ordered state in $d = 2$. An analysis of the specific heat behavior for the $d = 2$ case was done in Ref. [11] suggesting a different temperature dependance of the specific heat.

In order to calculate the temperature dependence of the resistivity we apply the self-consistent theory of fluctuations to the action given by Eq. (1). This can be done in the version of $1/n$ expansion (n being the number of components of the bosonic field ϕ) applied to the ϕ^4 action. Using the approximation $|\phi^4| \sim 2 < |\phi^2| > |\phi^2|$ the renormalized parameter $\delta(x)$ can be calculated from the following self-consistent equation:

$$\delta(x) \;=\; \delta_0(x) \;+\; \frac{u}{2}\left(\frac{n}{2}+1\right)\; k_B T \sum_{\omega_n} \int \frac{d^3\mathbf{k}}{(2\pi)^3}\; \frac{1}{\delta(x)+k^2+\frac{|\omega_n|}{\Gamma k}}\;. \tag{17}$$

The summation over the bosonic Matsubare frequencies on the second term in the right hand side (rhs) of Eq. (17) can be performed analytically leading to the following expression:

$$\delta(x) \;=\; \delta_0(x) \;+\; \frac{u}{2}\left(\frac{n}{2}+1\right) \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int_0^{\Gamma k} d\omega\; \coth\left(\frac{\omega}{2k_BT}\right) \frac{\frac{\omega}{\Gamma k}}{(\delta+k^2)^2+\left(\frac{\omega}{\Gamma k}\right)^2}\;. \tag{18}$$

The temperature dependence of the QPT parameter $\delta(x)$ can be extracted if we consider Eq. (18) at the QCP in order to eliminate the bare QPT parameter $\delta_0(x)$. Accordingly, we obtain

$$\begin{aligned}\delta(x) \;=\; & \frac{u}{2}\left(\frac{n}{2}+1\right) \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int_0^{\Gamma k} \frac{d\omega}{\pi} \left[\coth\left(\frac{\omega}{2k_BT}\right)-1\right] \frac{\frac{\omega}{\Gamma k}}{(\delta(x)+k^2)^2+\left(\frac{\omega}{\Gamma k}\right)^2} \\ & +\frac{u}{2}\left(\frac{n}{2}+1\right) \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int_0^{\Gamma k} \frac{d\omega}{\pi} \left[\frac{\frac{\omega}{\Gamma k}}{(\delta(x)+k^2)^2+\left(\frac{\omega}{\Gamma k}\right)^2} - \frac{\frac{\omega}{\Gamma k}}{k^4+\left(\frac{\omega}{\Gamma k}\right)^2}\right]\;. \end{aligned} \tag{19}$$

For $T = 0$ we will show that $\delta \ll T$ holds. In this approximation the first integral in the rhs of Eq. (19) becomes:

$$I_1 \;\cong\; 2k_BT \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int_0^{k_BT} \frac{d\omega}{\omega} \frac{\frac{\omega}{\Gamma k}}{A^2+\left(\frac{\omega}{\Gamma k}\right)^2}\;, \tag{20}$$

where $A = \delta(x) + k^2$. Performing the integral over ω we get:

$$\begin{aligned} I_1 \;&=\; \frac{\Gamma}{6\pi^3}\left(\frac{k_BT}{\Gamma}\right)^{4/3} \int_0^{\infty} dy\; \frac{\arctan y}{y^{4/3}} \\ &=\; C_1\,(k_BT)^{4/3}\;, \end{aligned} \tag{21}$$

where $y = k_BT/\Gamma k^3$; y_c, associated with the upper critical wave-vector k_c, has been substituted by infinity. C_1 is a constant. The second integral from the rhs of Eq. (19), I_2, is:

$$I_2 = \int_0^{\Gamma k} \frac{d\omega}{2\pi} \left[\frac{\frac{\omega}{\Gamma k}}{A^2+\left(\frac{\omega}{\Gamma k}\right)^2} - \frac{\frac{\omega}{\Gamma k}}{k^4+\left(\frac{\omega}{\Gamma k}\right)^2}\right]\;, \tag{22}$$

and can be performed if one introduce a new variable $y = k/\delta^{1/2}$ with an upper cut-off $y_c = k_c[\delta(x)]^{-1/2}$. The final result can be express as:

$$I_2 \;=\; -C_2\;\delta(x)\;. \tag{23}$$

Based on Eqs. (19), (21) and (23) we obtain for $\delta(x)$ the following temperature dependance:

$$\delta(x,T) \;=\; C(u,n)\;(k_BT)^{4/3}\;, \tag{24}$$

which is finite for $n \to \infty$. This result can be inverted in order to extract the concentration dependence of the critical temperature. If one consider that the temperature can be approximated by the critical value (T_c), and that in the first order $\delta(x) \approx \delta_0(x)$ we have

$$T_c(x) \sim (x - x_c)^{3/4}\;, \tag{25}$$

a result which is in agreement with the one discussed in Ref. [4].

The temperature dependance of the resistivity can be extracted as the imaginary part of the self-energy, obtained as a result of electrons interacting with the ferromagnetic fluctuations. In the one-loop approximation we have:

$$\Sigma(\mathbf{k}, i\omega_n) \;=\; g^2 k_BT \sum_{\mathbf{q}, i\omega_l} G(\mathbf{k}+\mathbf{q}; i\omega_n + i\omega_l)\; D(\mathbf{q}, i\omega_l)\;, \tag{26}$$

where g^2 is the coupling constant, $G(\mathbf{k}, i\omega_n)$ is the electronic Green function,

$$G(\mathbf{k}, i\omega_n) \;=\; \frac{1}{i\omega_n - \epsilon(\mathbf{k})}\;, \tag{27}$$

with $\omega_n = (2n+1)k_BT$, and $D(\mathbf{q}, i\omega_l)$ has the same form with $\chi(\mathbf{q}, \emptyset)$ given by Eq. (9) whit $\delta_0(x)$ replaced by $\delta(x)$. Performing the summation over ω_l in Eq. (26) we obtain:

$$\Sigma(\mathbf{k}, i\omega_n) = -\pi g^2 \sum_{\mathbf{q}} \int_{-\Gamma q}^{\Gamma q} \frac{dz}{\pi} \frac{\frac{z}{\Gamma q}}{A^2 + \left(\frac{z}{\Gamma q}\right)^2} \times [n_B(z) + n_F(z+\omega)]\, \delta[\omega + z - \epsilon(\mathbf{k}+\mathbf{q})] \ , \quad (28)$$

where $n_B(z)$ is the Bose function and $n_F(z)$ is the Fermi function. For $|z| \ll T$ we approximate $n_B(z) + n_F(z) \sim T/z$ and performing the analytical continuation $i\omega_n \rightarrow \omega + i\eta$ we calculate

$$-Im\, \Sigma^R(k_F, 0) \ \simeq \ \pi g^2 \sum_{\mathbf{q}} \int_{-\Gamma q}^{\Gamma q} \frac{dz}{\pi} \frac{\frac{T}{\Gamma q}}{A^2 + \left(\frac{z}{\Gamma q}\right)^2} \delta[z - \epsilon(\mathbf{k}+\mathbf{q})] \quad (29)$$

which leads to the following expression for the imaginary part of the self-energy

$$-\, Im\, \Sigma^R(k_F, 0) \ = \ \text{const} \frac{(k_BT)^3}{\delta(T)} \ . \quad (30)$$

Using now Eq. (24) we obtain the temperature dependence of the scattering time $1/\tau_{eff} = -Im\Sigma^R$ as:

$$\frac{1}{\tau_{eff}} \ = \ \text{const}\ (k_BT)^{5/3} \ , \quad (31)$$

which gives for the temperature dependent resistivity $\Delta\rho(T) = \rho(T) - \rho(0)$ the following behavior

$$\Delta\rho(T) \ \sim \ (k_BT)^{5/3} \ , \quad (32)$$

in agreement with the experimental data from Ref. [4].

IV. CONCLUSION

In conclusion, we presented a model for the ferromagnetic QPT driven by the magnetic impurities which polarize the Fermi liquid close to the Stoner instability. The result can be regarded as a generalization of the Doniach and Wohlfarth [7] mean-field model for the case of QPT. As a general result we conclude that the system presents a non-Fermi behavior in the critical region around a QCP. The non-Fermi character of the system is sustained by a $T \ln T$ behavior of the specific heat correction term, and by an electrical resistivity which presents a $T^{5/3}$ dependence. Both these results are in good agreement with the experimental data reported in Ref. [4]. A similar temperature dependance of the resistivity was obtained by Mathon[14] using a simple molecular field theory. However, despite a good agreement between the calculated and experimental values of the resistivity, Mathon calculations are not able to explain the non-Fermi behavior of the specific heat and do not take into account the quantum effects observed in Ni_xPd_{1-x}, making the model inadequate for the proper description of the QPT in this particular system. A phenomenological description of the QPT was also made by Lonzarich.[6] We also explain the concentration dependance of the critical temperature, our result being in good agreement with the experimental data.

We mention that this model is different from the BKNV [5] model, in which the spin susceptibility for the $d = 3$ case is considered as $\chi(q, \omega = 0) = \chi_0 + q^2 \ln(p_F/q)$. This form of $\chi(q)$ has been carefully analyzed by Millis [15] and at the present time new experimental data are needed for a confirmation of this spin susceptibility. Recently, Belitz and Kirkpatrick [16] have been reconsidered the QPT in the clean itinerant-electron ferromagnet. The coupling of the order parameter fluctuations to the soft fermionic fluctuations lead to a theory which is very different than theories based on the Hertz-Millis model. The main point in the new version of the BKNV theory is that the fluctuations can change the first order phase transition in a second order one. However, the occurrence of two dynamical exponents z and $\tilde{z}$ for the two kind of fluctuations makes the two theories very different, even if in any case the mean field behavior can explain the experimental data.

[†] Present Address: Universite Bordeaux 1, C.P.M.O.H., 33405 Talence, Cedex France
[‡] Present address: Department of Physics and Astronomy, The University of Iowa, Iowa City, IA 52242, USA.

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